## GCSE Maths - Probability

## Conditional Probability (Higher Only)

Notes

## WORKSHEET



## Conditional Probability

Conditional probability is the probability of an event occurring, given that another related event has already occurred. For example, the probability that it will rain this afternoon, given that it rained this morning, may be different to the probability that it will rain this afternoon, given that it was sunny this morning.

Conditional probability occurs when events are not independent. For independent events, conditional probability reduces to normal probability since the events do not have an effect on one other.

The notation $\mathrm{P}(\mathrm{A} \mid \mathrm{B})$ means the probability of A occurring given that B has already occurred.
The formula for conditional probability is: $P(A \mid B)=\frac{P(A \cap B)}{P(B)}$

## Two-Way Tables

Two-way tables will show the frequency of specific categories of data. Conditional probability from two-way tables can be calculated when an event occurs given it is in a specific column or row.

Example: The table shows information about students in a class. A girl is chosen at random from the class. Find the probability that this girl walks to school.

|  | Walk | Public <br> Transport | Car | Total |
| :---: | :---: | :---: | :---: | :---: |
| Boy | 10 | 5 | 9 | 24 |
| Girl | 15 | 2 | 9 | 26 |
| Total | 25 | 7 | 18 | 50 |
| There are 26 girls in the class |  |  |  |  |

As we already know that this student is a girl, the probability is out of 26 because there are 26 girls in the class.

Out of the 26 girls, 15 girls walk to school.

$$
P(\text { Girl who walks to school })=\frac{15}{26}
$$

## Venn Diagrams

Venn Diagrams organise information about events and their probability. Conditional probability can be calculated from Venn diagrams when an event occurs given that it is in a specific region.

Example: Calculate $P\left(A \mid B^{\prime}\right)$ from the following Venn diagram. Here, $B^{\prime}$ means "not $\mathrm{B}^{\prime}$.

$P\left(A \mid B^{\prime}\right)$ means the probability of $A$ occurring given that it is not in the region of $B$.
There is a frequency of 20 in $B^{\prime}$, because $15+5=20$ are not included in the circle of $B$.
We already know that the event occurs in this region, so the probability is out of 20 .
Out of the 20 in $B^{\prime}, 15$ also occur in $A$.

$$
\begin{aligned}
& P\left(A \mid B^{\prime}\right)=\frac{15}{20} \\
& P\left(A \mid B^{\prime}\right)=\frac{3}{4}
\end{aligned}
$$

Further information on Venn Diagrams can be found in:

- Maths GCSE Revision Notes - Probability - Enumeration, Venn Diagrams, Tree Diagrams and Tables


## Tree Diagrams

Tree Diagrams show the different outcomes that can occur in an event. Like Venn Diagrams and two-way tables, a question may ask you to calculate the probability of an event occurring, given that another event has occurred. You can follow the branches of a tree to work out whether events are related, and then combine the probabilities along the branches that you have followed.

This event may be the related to the first event. For example, you may be asked to find the probability of picking a red ball out of a bag and then not replacing it for the second event.

Example: There are 6 red balls and 4 blue balls in a bag. Toni picks two balls from the bag. Calculate the probability that two red balls are selected.

This question implies conditional probability as the first ball selected is not replaced. The question is asking you to calculate the probability that a red ball will be picked after a red ball has already been taken out.

Drawing a tree diagram:


The probability of a red ball being picked out first is $\frac{6}{10}$.
The probability of a red ball being picked out again is $\frac{5}{9}$, because there is one less (red) ball in the bag then the first trial.

The probability of 2 red balls being picked is $\frac{6}{10} \times \frac{5}{9}=\frac{30}{90}$.

$$
P(\text { Picking } 2 \text { red balls })=\frac{1}{3}
$$

Further information on Tree Diagrams can be found in:

- Maths GCSE Revision Notes - Probability - Table of Outcomes and Frequency Trees


## Conditional Probability - Practice Questions

1. A student who doesn't have a pet is chosen at random. Calculate the probability that they are a boy.

|  | Cat | Dog | No Pet | Total |
| :---: | :---: | :---: | :---: | :---: |
| Boy | 14 | 3 | 10 |  |
| Girl | 11 | 4 | 8 |  |
| Total |  |  |  | 50 |

2. A boy is chosen at random. Calculate the probability that he studies French.

|  | French | German | Spanish | Total |
| :---: | :---: | :---: | :---: | :---: |
| Boy |  | 2 |  |  |
| Girl | 7 |  | 12 |  |
| Total | 10 | 5 | 25 |  |

3. Calculate $P\left(A \mid C^{\prime}\right)$ for the following Venn diagram.

4. There are 2 yellow, 3 red and 6 blue marbles in a bag. Find the probability that a yellow marble is chosen after a yellow marble is taken out.
